Towards Bayesian Subject Model

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Simple subject model

$$U_{ij} = \psi_j + \Delta_i + \upsilon_i X + \phi_j Y$$

$$X, Y \sim N(0, 1)$$

Parameters

$$\theta = (\psi_j, \Delta_i, \upsilon_i, \phi_j,)$$

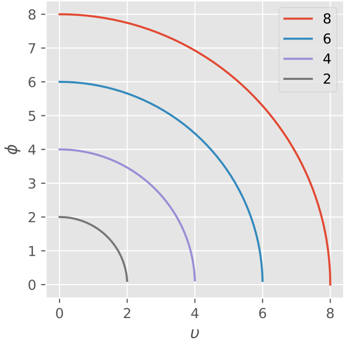
Definition

Maximum likelihood estimator

$$\hat{\theta} = \arg\max_{\theta} P(u|\theta)$$

Problems

1. $U_{ij} \in \{1, 2, 3, 4, 5\} \sim N(\psi_j + \Delta_i, \sqrt{\upsilon_i^2 + \phi_j^2})$
2. Non unique solutions: how to partition variance among testers and PVSs?



Solutions

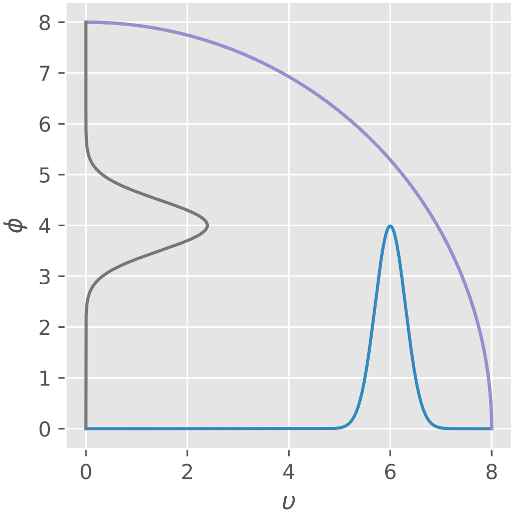
1. $U_{ij} \sim Q(N(\psi_j + \Delta_i, \sqrt{\upsilon_i^2 + \phi_j^2}))$, $Q()$ - a quantizer (e.g ceil)
2. Use prior knowledge (expert, domain specific, etc) about parameters expressed as distribution.

Example

Bayesian model

1. $\upsilon \sim Gamm(\alpha_1, \beta_1)$
2. $\phi \sim Gamm(\alpha_2, \beta_2)$

Priors



Bayesian subject model - not yet

Theorem (Bayes)

$$P(\theta|u) = \frac{P(u,\theta)}{P(u)} = \frac{P(u|\theta)P(\theta)}{P(u)}$$

Definition

Maximum a'posteriori estimator (MAP)

$$\hat{\theta} = \arg\max_{\theta} P(\theta|u)$$

MLE → MAP

$$\log P(\theta|u) = \log P(u|\theta) + \log P(\theta) - \log P(u)$$

Just add regularization given by log prior $\log P(\theta)$.

Tensorflow Probability

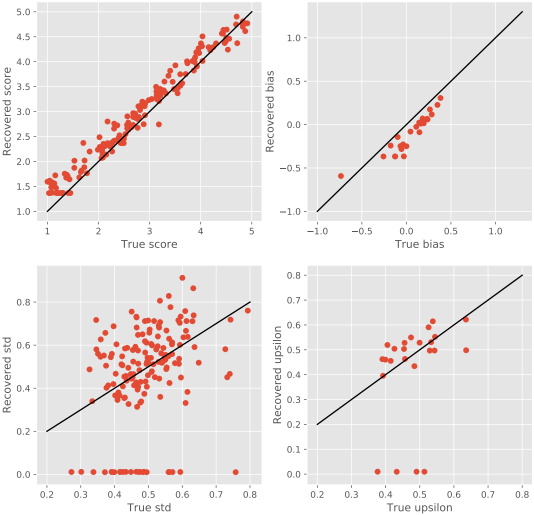
1. Based on TensorFlow, R like, GPU Accelerated.
2. Rich library of distributions (Normal, **QuantizedDistribution**).
3. Probabilistic programming language for model building.
4. Optimizers from TensorFlow for MAP, MLE ($\arg\min_\theta$)
5. Designed for Bayesian Inference (**MCMC**, tbd...).

Example

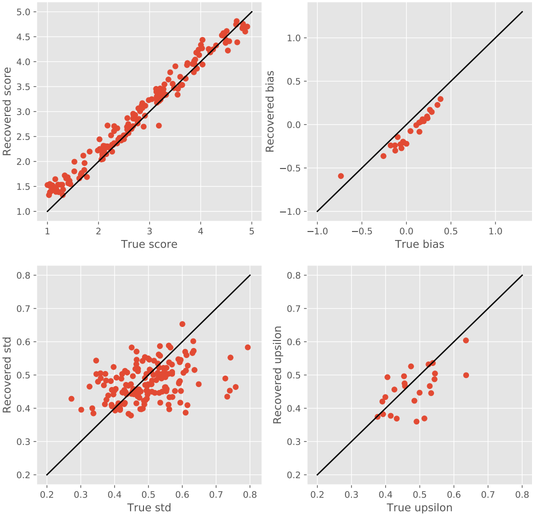
`https://goo.gl/G4XR1C`

Results

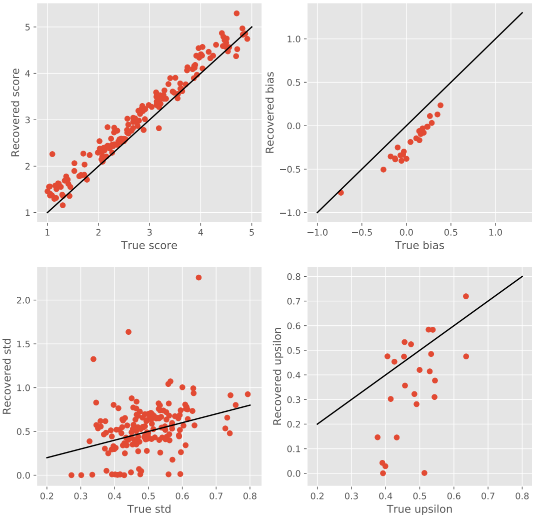
MLE - continuous



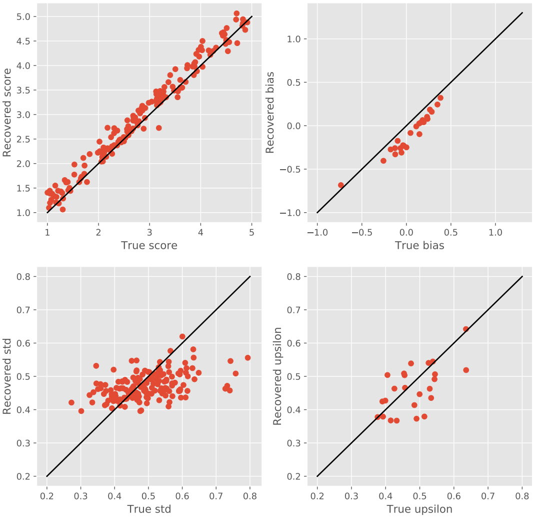
MAP - continuous



MLE - quantized



MAP - quantized



Summary

1. MAP as a simple extension of MLE
2. TensorFlow Probability allows fitting quantized distribution
3. Optimization poses numerical problems
4. Next step: MCMC for full posterior distribution (confidence intervals)