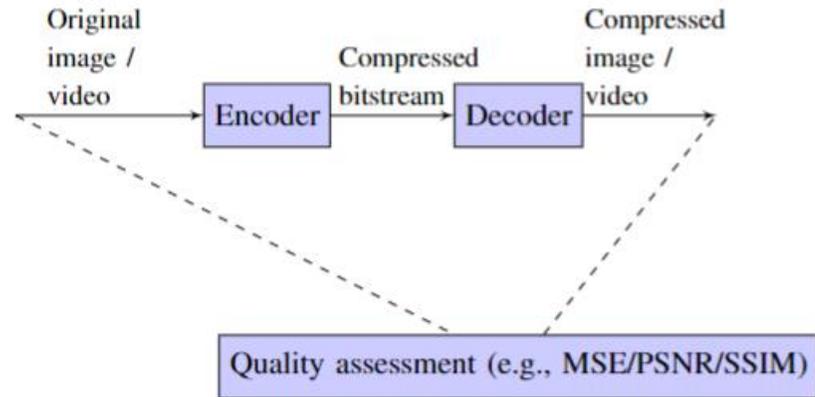


A simple relationship between SSIM and PSNR for DCT-based compressed images and video: SSIM as content-aware PSNR

Prof Maria Martini
Kingston University London

VQEG meeting 15/12/2022

Reminder

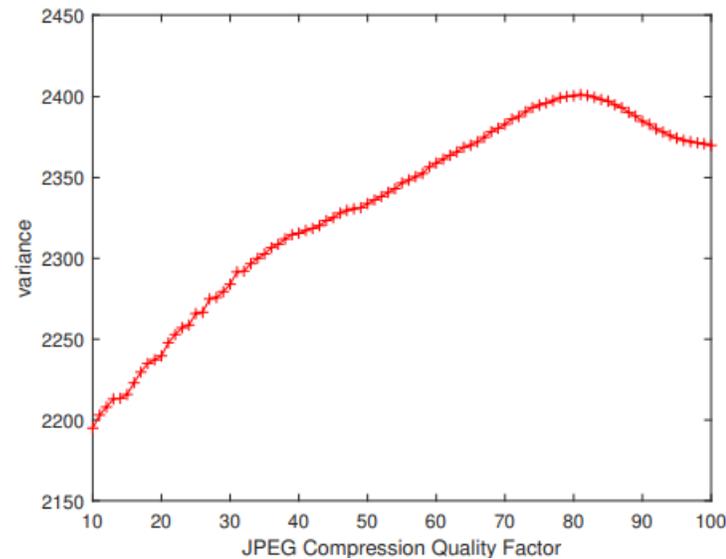
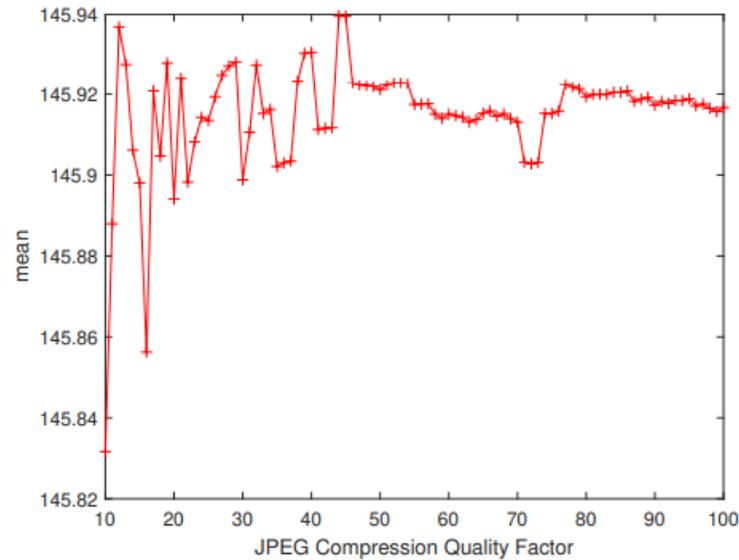


$$PSNR = 10 \log_{10} \frac{(2^b - 1)^2}{MSE}$$

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

Assumptions for DCT compressed images / video

$$\mu_e = 0, \mu_x = \mu_y \text{ and } \sigma_x^2 = \sigma_y^2$$



Relationship derivation

With these assumptions: [reminder: $SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$]

$$SSIM(x, y) = \frac{(2\cancel{\mu_y^2} + C_1)(2\sigma_{xy} + C_2)}{(2\cancel{\mu_y^2} + C_1)(2\sigma_y^2 + C_2)}$$

$$SSIM(x, y) = \frac{(2\sigma_{xy} + C_2)}{(2\sigma_y^2 + C_2)}$$

$$e = x - y$$

$$\sigma_e^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}$$

$$2\sigma_{xy} = \sigma_x^2 + \sigma_y^2 - \sigma_e^2 = 2\sigma_y^2 - \sigma_e^2$$

$$SSIM(x, y) = \frac{(2\sigma_y^2 - \sigma_e^2 + C_2)}{(2\sigma_y^2 + C_2)} = 1 - \frac{\sigma_e^2}{(2\sigma_y^2 + C_2)}$$

With the previous assumption $\mu_e = 0$ we have

$$\sigma_e^2 = \mathcal{E}(e^2) = MSE$$

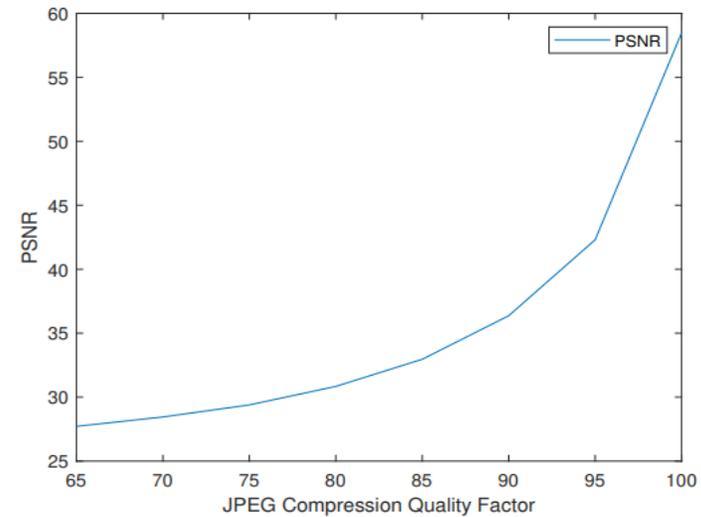
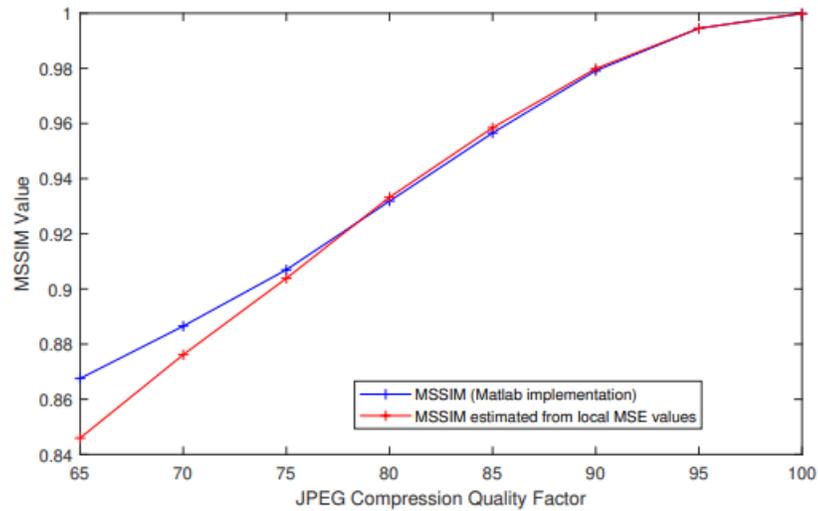
$$SSIM(x, y) = 1 - \frac{MSE}{(2\sigma_y^2 + C_2)}$$

$$PSNR = 10 \log_{10} \frac{(2^b - 1)^2}{MSE} \quad SSIM(x, y) = 1 - \frac{\frac{(2^b - 1)^2}{10^{PSNR/10}}}{(2\sigma_y^2 + C_2)}$$

$$MSSIM(X, Y) = \frac{1}{M} \sum_j SSIM(x_j, y_j)$$

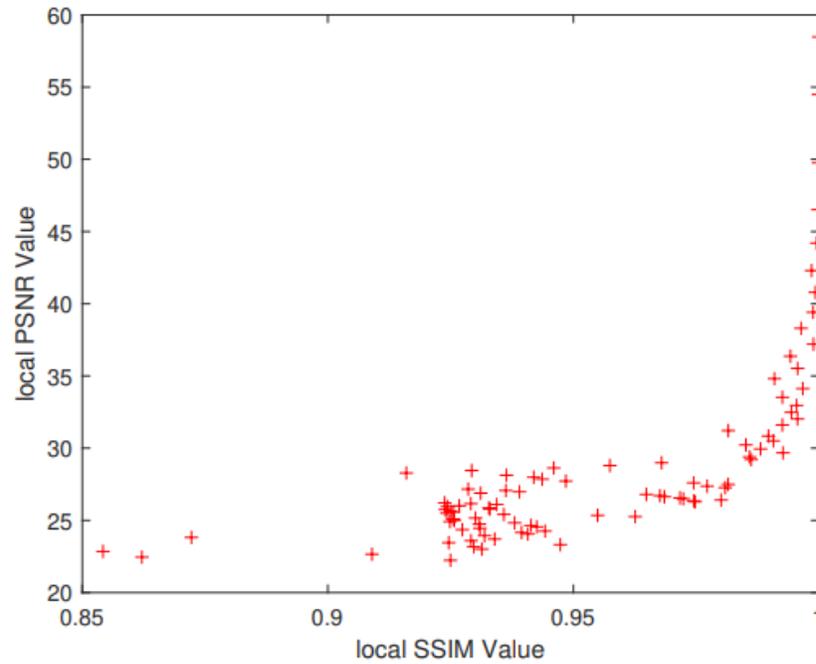
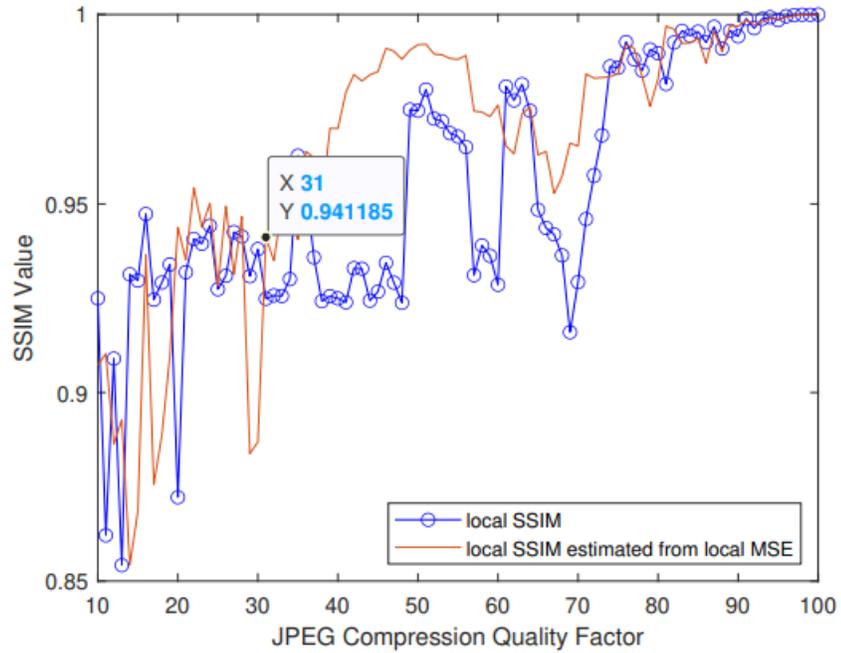
$$MSSIM(X, Y) = \frac{1}{M} \sum_j \left[1 - \frac{\frac{(2^b - 1)^2}{10^{PSNR_j/10}}}{(2\sigma_{y_j}^2 + C_2)} \right]$$

Example results

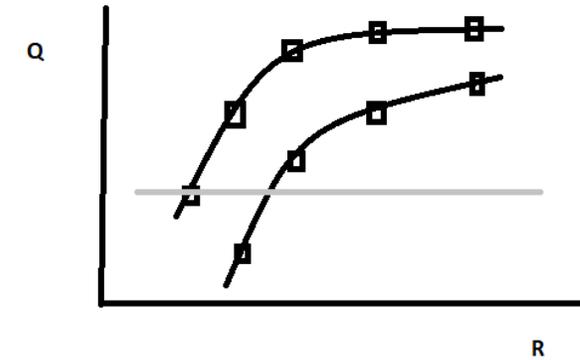
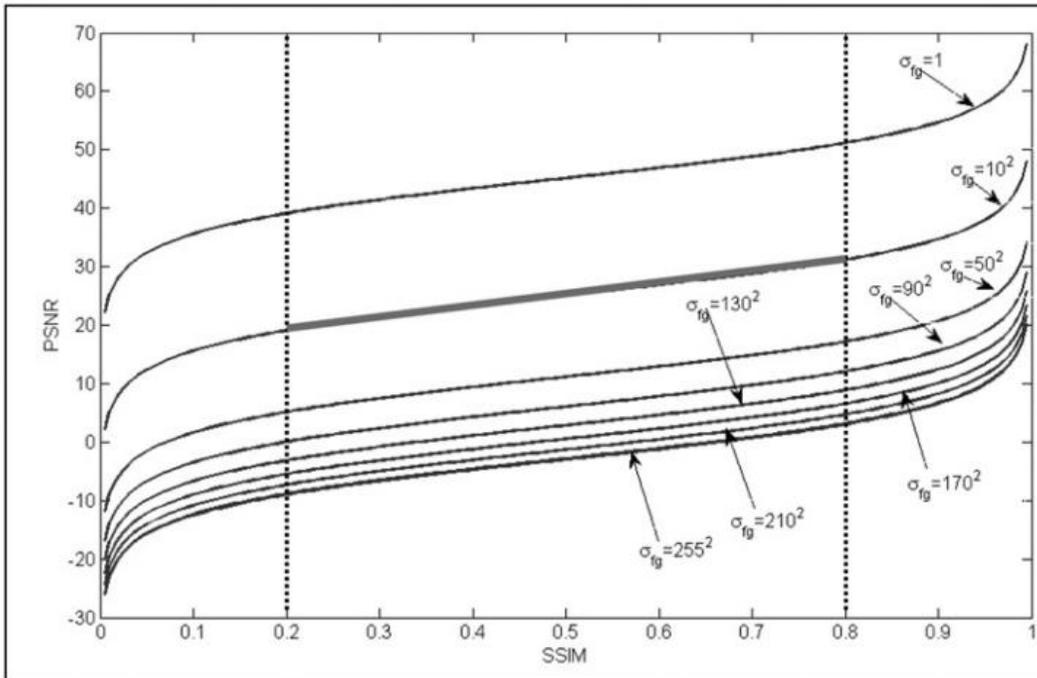


$$\text{MSSIM}(X, Y) = \frac{1}{M} \sum_j \left[1 - \frac{(2^b - 1)^2}{10^{\frac{PSNR_j}{10}} (2\sigma_{yj}^2 + C_2)} \right].$$

Local results



Prior work



Hore, A., & Ziou, D. (2010). Image quality metrics: PSNR vs. SSIM. In *2010 20th international conference on pattern recognition* (pp. 2366-2369). IEEE.

Horé, A., & Ziou, D. (2013). Is there a relationship between peak-signal-to-noise ratio and structural similarity index measure? *IET Image Processing*, 7(1), 12-24.

Conclusion

- Simple relationship between SSIM and PSNR
- Only depending on a simple content factor (image variance)
- Assumptions reasonably good for quality range of typical interest
- Useful simplification for optimization problems
- Example applications include no-reference estimation of SSIM from PSNR

Associated paper:

Martini, Maria (2022): On the relationship between SSIM and PSNR for DCT-based compressed images and video: SSIM as content-aware PSNR. TechRxiv. Preprint.

<https://doi.org/10.36227/techrxiv.21725390.v1>

[Link](#)

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