Problem with Continuous Model

Lucjan Janowski, Krzysztof Rusek, Bogdan Ćmiel

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Continuous Model

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Let us consider 5 point scale so we have censoring (clipping) and discretization. So the final answer is:

$$U_{ij} = \mathrm{Round}(U_{ij}) \sim F$$

Simplification

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So we can validate what is the relation between ψ_o and ψ_u.

$$O_{ij} \sim \mathcal{N}(\psi_o, \sigma_o)$$

$$P(U_{ij} = k) = \begin{cases} \int_{-\infty}^{1.5} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k = 1 \\ \int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k \in \{2,3,4\} \\ \int_{4.5}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k = 5 \end{cases}$$

Knowing $P(U_{ij} = k)$ we can calculate ψ_u and plot function $\psi_u(\psi_o)$ it should be $y = x$.

Python Code

```
inputTQ = np.arange(1.01, 4.99, 0.01)
sigma = 0.1

p1 = norm.cdf(1.5, loc = inputTQ, scale = sigma)
p2 = norm.cdf(2.5, loc = inputTQ, scale = sigma) -
    norm.cdf(1.5, loc = inputTQ, scale = sigma)
p3 = norm.cdf(3.5, loc = inputTQ, scale = sigma) -
    norm.cdf(2.5, loc = inputTQ, scale = sigma)
p4 = norm.cdf(4.5, loc = inputTQ, scale = sigma) -
    norm.cdf(3.5, loc = inputTQ, scale = sigma)
p5 = 1 - norm.cdf(4.5, loc = inputTQ, scale = sigma)

outputTQ = p1 + 2*p2 + 3*p3 + 4*p4 + 5*p5
plt.plot(inputTQ, outputTQ)
```

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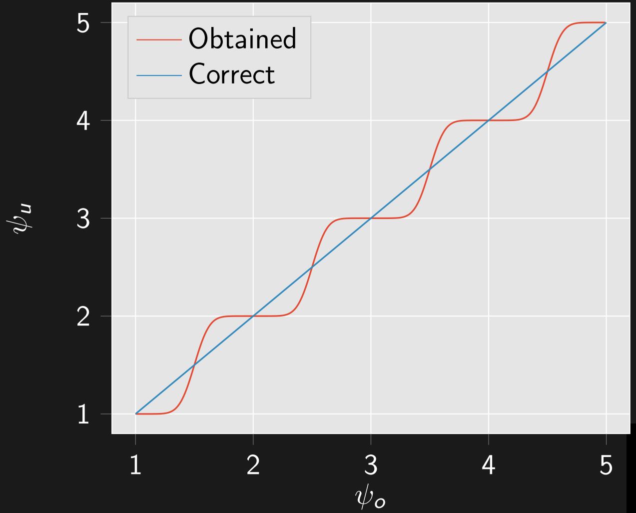
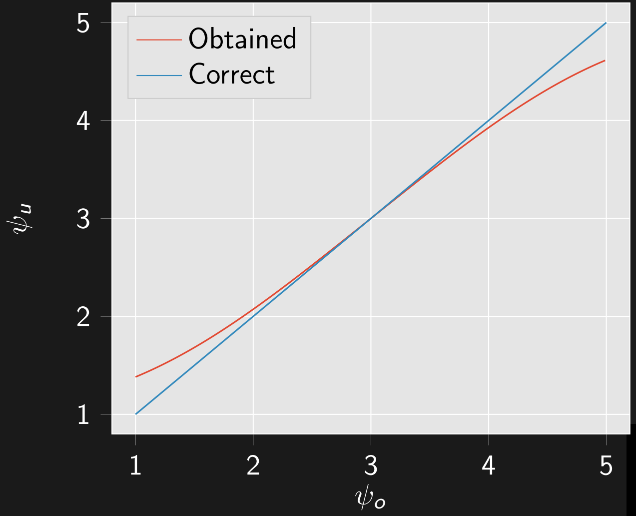
Let as assume that $\sigma_o = 0.1$.

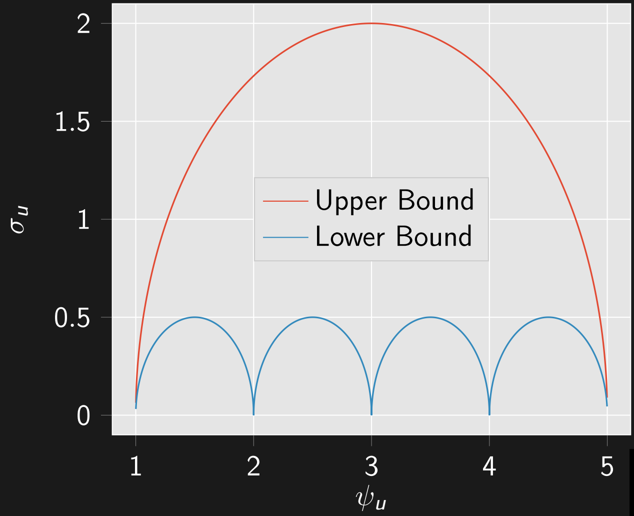


Large Standard Deviation

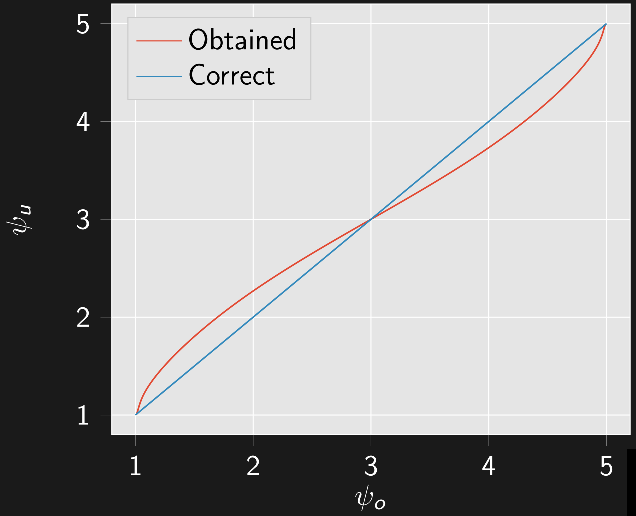
Let as assume that $\sigma_o = 1$.



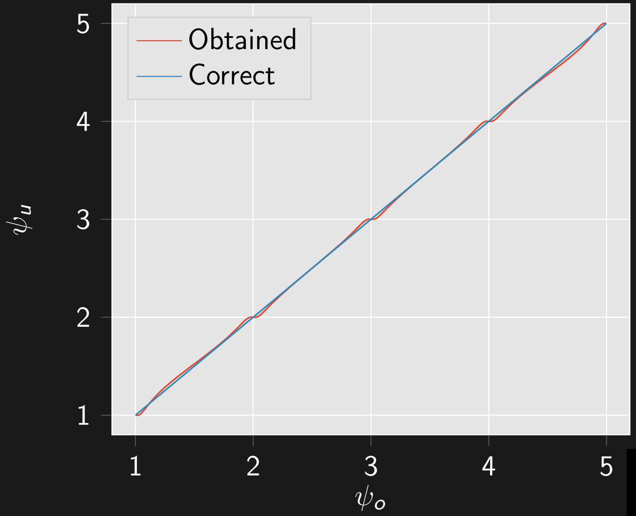
Can We Set σ_o to Any Value?



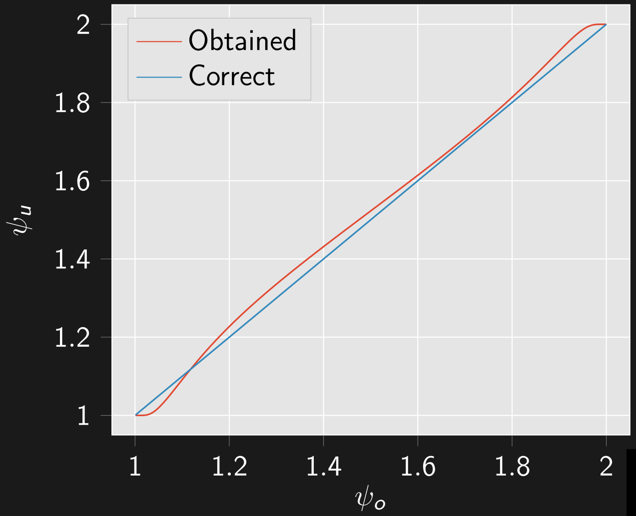
Maximum Std



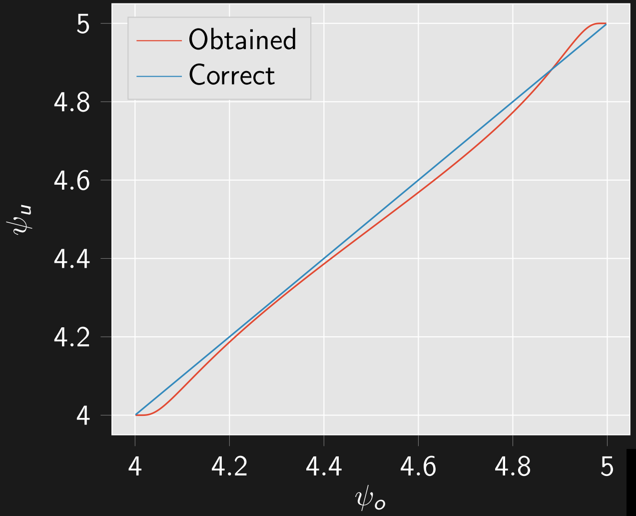
Minimum Std



Closer Look $\psi_o \in (1, 2)$



Closer Look $\psi_o \in (4, 5)$



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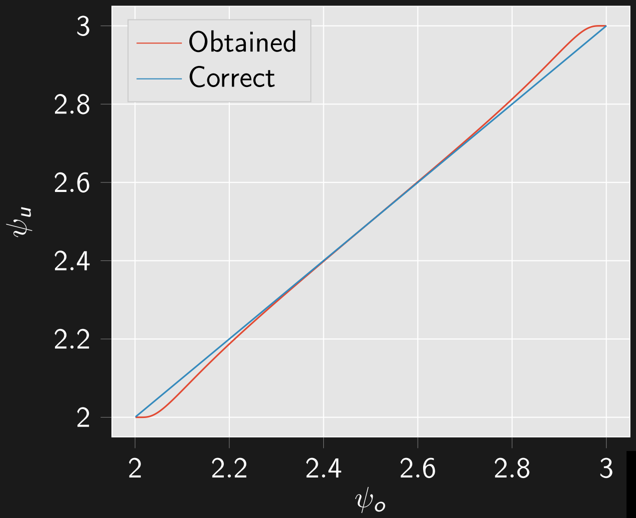
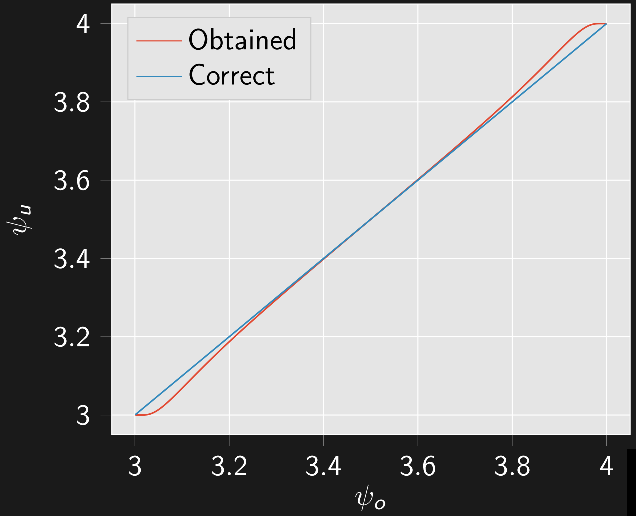
Closer Look $\psi_o \in (2, 3)$



Closer Look $\psi_o \in (3, 4)$



Maximum Std for 0-100 Scale